A Formal Model for Representing Point Trajectories in Two-Dimensional Spaces

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Abstract. Modelling moving points is a subject of interest that has attracted a wide range of spatio-temporal database research. Efforts so far have been oriented towards the development of database structures and query languages. The preliminary research presented in this paper introduces a formal analysis of spatio-temporal trajectories, where the objective is to complement current proposals by a categorization of the underlying processes that characterize moving points. The model introduced identifies the semantic exhibited by point versus point, point versus line and point versus region trajectories.

1 Introduction

The integration of the temporal dimension within GIS is an area that has attracted a wide range of spatio-temporal databases research [1], [2]. This is largely favoured by the constant evolution of geo-referencing systems such as transponders for airplanes and ships, and telecommunication and GPS for terrestrial navigation. Applications range from monitoring to simulation systems, prediction and planning studies where the objective is to control and analyse fast changing phenomena such as urban or navigation traffic and transportation systems. Achievements and prototypes realised so far include developments of spatio-temporal data types and query languages [3], [4], [5], [6] and physical storage structures [7], [8]. These research achievements call for further development of exploration interfaces and languages that will help to identify processes, trends, and patterns that characterize the dynamic properties of spatio-temporal applications. In particular, such systems should help traffic and transportation planners to observe and understand the evolution of a given dynamic system at different levels of granularity.

The objective of the research presented in this paper is to explore and develop a trajectory manipulation model that supports not only the representation of mobile trajectories, but also an intuitive data manipulation language that facilitates the understanding of the underlying behaviours, processes and patterns exhibited by moving points. Our objective is not to develop a new spatial query language, but rather to explore the semantics revealed by point trajectories in space and time, and to identify a topological language that allows for a derivation of a complete set of orthogonal and plausible processes.

The remainder of the paper is organised as follows. Section 2 introduces basic principles of trajectory modelling, using either absolute or relative views

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of space, and motivates the need for an integration of absolute and relative representations. Section 3 develops a formal model of mobile trajectories, that is, points moving in a two-dimensional space with respect to points, lines and regions, and their continuous transitions. Finally section 4 concludes the paper and draws some conclusions.

2 Modelling Background

Recent developments in Geographical Information Science tend to provide some alternative models of space that surpass the conventional Euclidean vision of space. Spatial cognition, naive geography are some of the domains that contribute to the emergence of cognitive representations of space ([9], [10]) particularly in the case of "egocentric" views where the objective is to model and materialize space the way it is perceived from a mobile observer active in the environment [11].

Such aspects in the representation of space directly relate to the main difference between an absolute and a relative representation, where in the former the origin and system of reference are fixed, while in the latter they depend of the location of the observer, or the observers when multiple points of view are required. In fact, absolute and relative representations provide complementary views, they can be also partially derived one from the other. Absolute representations are very much adapted and applied to the analysis of global patterns and trends, using conventional spatial data analysis and statistics, while relative representations have not been very much used, to the best of our knowledge, for data analysis and mining. Taking a relative view of space, it might be possible to analyse the behaviour of a moving object in space, with respect to other regions of space with which this object is in interaction. However, this implies to explore in further details the semantics revealed by a trajectory, when perceived from a relative point of view.

At the perception level, cognitive and spatial reasoning studies [12], [13], [14], [15] have shown than distance and speed differences are amongst the relationships and processes that are intuitively perceived and understood by human beings when acting in the environment and perceiving other mobile actors. This leads us to explore and design a trajectory data model, where distance and speed differences are modelled over time. One of our research objectives is to explore to which degree such a model completes the conventional Euclidean view of a spatial trajectory. From a formal point of view, this should lead to design a model made of a complete set of physically plausible and orthogonal process primitives, that also makes sense as far as possible at the cognitive level.

3 Trajectory Modelling

Let us study the trajectory of a moving point, modelled relatively with respect to some parts of space (i.e. point, line or region). Let us consider a two-dimensional space, a moving region A and a moving point B. We say that the moving region

A is the reference of the relative view and then the origin of the relative frame of reference, the moving point B being the target (or the reverse). The two-dimensional relative view is given by a coordinate system where the abscissa axis represents the speed difference between the target point and the origin region. An object materialized by either a line or a region is considered having an homogeneous speed and not deformed through time. The difference of speed ($\Delta v_{A,B}(t)$) between a target point and an origin region (or the reverse) is given as follows :

$$\Delta v_{A,B}(t) = \begin{cases} +\sqrt{(v_{x_A}(t) - v_{x_B}(t))^2 + (v_{y_A}(t) - v_{y_B}(t))^2} \\ \text{If } \|\overrightarrow{v_A}(t)\| \geq \|\overrightarrow{v_B}(t)\| \\ -\sqrt{(v_{x_A}(t) - v_{x_B}(t))^2 + (v_{y_A}(t) - v_{y_B}(t))^2} \\ \text{Otherwise} \end{cases} \tag{1}$$

Per convention, we say that when the target point B is slower than the origin region A, $\Delta v_{A,B}(t)$ is negative, positive on the contrary. The ordinate axis represents, if one of the objects is a region, the minimum Euclidian distance between the region and the point when the point is outside the region, the negative of this value when the point is inside the region, and the null value when the point is in the boundary of the region. When the origin and the target are both points or a line and a point, the Euclidean distance is considered. These distance are given as follows:

$$d_{A\times B}(t) = \begin{cases} +(\sqrt{(x_A(t) - x_B(t))^2 + (y_A(t) - y_B(t))^2}) \\ \text{If the point } B \text{ is outside the region or not within the line A, or} \\ A \text{ and } B \text{ are two points ;} \\ \text{where } (x_A(t), y_A(t)) \text{ denotes the coordinates of the point of } A \text{ the} \\ \text{closest to } B \text{ if } dim(A) = 2 \text{ or } dim(A) = 1 \text{, the coordinates of the} \\ \text{point otherwise} \\ -(\sqrt{(x_A(t) - x_B(t))^2 + (y_A(t) - y_R(t))^2}) \\ \text{If the point } B \text{ is into the region } A \\ \text{where } (x_A(t), y_A(t)) \text{ denotes the coordinates of the point in the} \\ \text{boundary of } A \text{ the closest to } B \end{cases} \tag{2}$$

A two-dimensional representation space is derived from the difference of speed and distance values. This two-dimensional space constitutes a modelling support for the exploration of (1) different dynamic states, (2) possible transitions between them and (3) physically plausible transitions. We hereafter successively study these three aspects.

Firstly, this representation space gives a partition that supports the characterization of the different spatio-temporal configurations, and the underlying processes than can be derived from moving point trajectories (Figure 1):

- $K = \{(\Delta v(t), d(t)) | d(t) > 0 \wedge \Delta v(t) < 0\}$
- $L = \{(\Delta v(t), d(t)) | d(t) > 0 \wedge \Delta v(t) > 0\}$
- $U = \{(\Delta v(t), d(t)) | d(t) < 0 \wedge \Delta v(t) < 0\}$
- $V = \{(\Delta v(t), d(t)) | d(t) < 0 \wedge \Delta v(t) > 0\}$

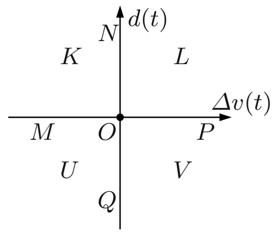

Fig. 1. Partition of the relative space

- $O = \{(\Delta v(t), d(t)) | d(t) = 0 \wedge \Delta v(t) = 0\}$
- $M = \{(\Delta v(t), d(t)) | d(t) = 0 \wedge \Delta v(t) < 0\}$
- $N = \{(\Delta v(t), d(t)) | d(t) > 0 \wedge \Delta v(t) = 0\}$
- $P = \{(\Delta v(t), d(t)) | d(t) = 0 \wedge \Delta v(t) > 0\}$
- $Q = \{(\Delta v(t), d(t)) | d(t) < 0 \wedge \Delta v(t) = 0\}$

Each of these spatio-temporal configurations denotes a spatio-temporal state of a target point B, with respect to a reference region A. We say that a state is valid over an interval of time $i \in I$, I being the set of temporal intervals.

Secondly, a state can be interpreted as a continuous event whose evolution is related to the way the two spatial dimensions considered evolve, i.e., distance and speed differences. Continuous transition between states can be formally studied using the notion of conceptual neighbours [16], [17] as distance and speed over time are continuous functions [18]. More formally, a continuous transition is defined as follows [16]:

Definition 1. ***Continuous transition***
A continuous transition between two spatio-temporal states materializes a continuous change without any intermediary state.

The set formed by such conceptual neighbours relations is defined as in [16], where such transitions are orthogonal and form a complete set. Formally, two path-connected sets are conceptual neighbourhoods if there are some continuous transitions between them [19]. This allows us to refine the notion of continuous transitions between two given states:

Definition 2. ***Continuous transitions versus path-connected sets***
There is a continuous transition between two states if their union is path-connected.

These definitions allowed us to identify the possible continuous transitions between the states identified [20]. These continuous transitions concern the changes of states from K to M, K and O, K and N, L and P, L and N, L and O, O and M, O and N, O and P, U and M, U and O, U and Q, V and P, V and O, V and Q and O and Q, and their reverses (Figure 2). One can remark that while some transitions between two states are continuous, other not (e.g. N and P or K and L). There is a continuous transition between two states if the

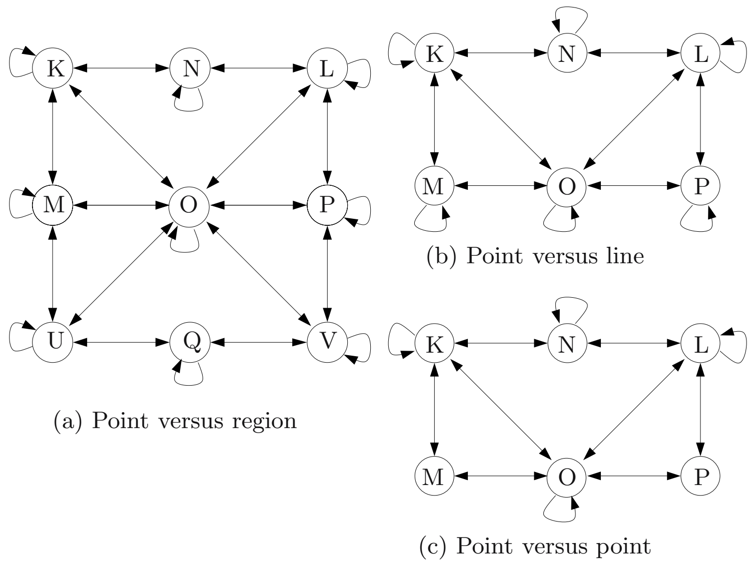

Fig. 2. Continuous transitions

set composed by the two states is a connected set. Conversely, if a set composed by two states is disconnected [21], then there is no continuous transition between them.

Thirdly, let us study the soundness of these continuous transitions as previously defined in [22], and the constraints that materialize them. We still consider a target point B and a referent region, line or point A.

Constraint 1 : ***Changes of states***
Let us assume that the speeds of on origin region A and a target point B are constant and equal, and that A and B are located at a given constant distance. This represents the case where the point B follows the region A (or the reverse). If that distance is not null, B is in the state N or Q, otherwise in the state O. A change of distance implies a change of difference speed, then a change of state N (resp. Q) or O to state K or L (resp. U or V).

By a straight application of this constraint, it is immediate to derive that there is no sound continuous transition between the states N and O, and Q and O.

Constraint 2 : ***Stable states***
Let us consider B in a stable state over time with respect to A. If B and A are materialized by points, then a constant and null value of distance over this state should be valid for a given instant only, but not over an interval of time I. Also the distance $d_{A,B}(t)$ cannot be null for any instant $t \in I$ if the difference of speed $\Delta v_{A,B}(t)$ is not null.

Applying constraint 2, the states M and P cannot last over time when the target and origin objects are points. It is also worth noting that some states can be

Table 1. Change of speed processes (with $t_0 < t_1$)

1	$t_0 : v_A > v_B$ $t_1 : v_A = v_B$	Deceleration of A or/and acceleration of B to reach the same speed at t_1
2	$t_0 : v_A = v_B$ $t_1 : v_A > v_B$	Acceleration of A or/and deceleration of B standing from the same speed at t_0
3	$t_0 : v_A < v_B$ $t_1 : v_A = v_B$	Acceleration of A or/and deceleration of B to reach the same speed at t_1
4	$t_0 : v_A = v_B$ $t_1 : v_A < v_B$	Deceleration of A or/and acceleration of B standing the same speed at t_0
5	$t_0 : v_A > v_B$ $t_1 : v_A > v_B$	A is faster than B
6	$t_0 : v_A = v_B$ $t_1 : v_A = v_B$	A and B have the same speed
7	$t_0 : v_A < v_B$ $t_1 : v_A < v_B$	B is faster than A

stable although some others cannot. The application of constraints 1 and 2 make possible the stability over time of states K, N, L and O while states M and P are instantaneous states, only. If the origin or referent objects are materialized by a line (or a region), the states K, N, L, M, O, P, U, Q and V are stable states. Applying constraints 1 and 2, a complete and sound set of continuous transitions can be derived for all representations. Applied to the modelling of the relative trajectories of two mobile objects (i.e. point and region, point and line and two points) A and B, possible continuous transitions are shown in Figure 2. Note that point versus point and point versus line cases don't have the cases where the difference of speed is negative.

There are also no continuous transitions between O and Q, and O and N as these changes of state contradict constraint 1. Applying constraints 1 and 2, and for interactions between a point and a region, a set of twenty eight continuous transitions between states and nine stable states is identified and shown in table 2. For interactions between a point and a line, there are sixteen continuous transitions between states and six stable states (Table 3). The set of continuous transitions representing the interactions between two points is the set of continuous transitions for interactions between point and line without transitions $P \rightarrow P$ and $M \rightarrow M$ (Constraint 2)(Table 3). Table 2 and 3 materialize sound continuous transitions valid for a change of state between two time-stamps, that is, time instants denoted t_0 and t_1 with $t_0 < t_1$. The semantics of the processes that imply a change of speed are ilustred in table 1.

It is worth noting that each continuous transition tends to represent the semantic of a specific and relative process of a target object with respect to an origin object (Table 2). One can also remark that the continuous transitions exhibited by the model correspond to processes that can be discriminated by a natural language expression which has the advantage of being unambiguous and relatively short.

Table 2. Continuous transitions versus dynamic processes (with $t_0 < t_1$)

	From outside the region to the boundary	From the boundary, to outside the region	Outside the region	On the boundary	In the region	From inside the region to the boundary	From the boundary to inside the region
	$t_0 : d > 0$ $t_1 : d = 0$	$t_0 : d = 0$ $t_1 : d > 0$	$t_0 : d > 0$ $t_1 : d > 0$	$t_0 : d = 0$ $t_1 : d = 0$	$t_0 : d < 0$ $t_1 : d < 0$	$t_0 : d < 0$ $t_1 : d = 0$	$t_0 : d = 0$ $t_1 : d < 0$
1	$L \rightarrow O$	$\varnothing$	$L \rightarrow N$	$P \rightarrow O$	$V \rightarrow Q$	$V \rightarrow O$	$\varnothing$
2	$\varnothing$	$O \rightarrow L$	$N \rightarrow L$	$O \rightarrow P$	$Q \rightarrow V$	$\varnothing$	$O \rightarrow V$
3	$K \rightarrow O$	$\varnothing$	$K \rightarrow N$	$M \rightarrow O$	$U \rightarrow Q$	$U \rightarrow O$	$\varnothing$
4	$\varnothing$	$O \rightarrow K$	$N \rightarrow K$	$O \rightarrow M$	$Q \rightarrow U$	$\varnothing$	$O \rightarrow U$
5	$L \rightarrow P$	$P \rightarrow L$	$L \rightarrow L$	$P \rightarrow P$	$V \rightarrow V$	$V \rightarrow P$	$P \rightarrow V$
6	$\varnothing$	$\varnothing$	$N \rightarrow N$	$O \rightarrow O$	$Q \rightarrow Q$	$\varnothing$	$\varnothing$
7	$K \rightarrow M$	$M \rightarrow K$	$K \rightarrow K$	$M \rightarrow M$	$U \rightarrow U$	$U \rightarrow M$	$M \rightarrow U$

We illustrate the potential of the relative modelling approach using some simplified examples that support a schematic representation of the different continuous transitions and stability states supported by the trajectory model. These correspond to common dynamic processes exhibited by the behaviour of moving objects in space and time. Figure 3 presents two evolving configurations where a target object is getting close of the origin object at an increasing speed (Figure 3.b), while in Figure 3.d, a mobile object is getting close, but with a difference of speed that decreases. Taking an absolute view, these behaviours are ambiguous as changes of speed are not represented, whereas with the relative representation of difference of speeds, behaviours are unambiguous.

Table 3. Continuous transitions and dynamic behaviours (with $t_0 < t_1$)

	Connection	Separation	Same location	Different locations
	$t_0 : d > 0$ $t_1 : d = 0$	$t_0 : d = 0$ $t_1 : d > 0$	$t_0 : d = 0$ $t_1 : d = 0$	$t_0 : d > 0$ $t_1 : d > 0$
1	$L \to O$	$\varnothing$	$P \to O$	$L \to N$
2	$\varnothing$	$O \to L$	$O \to P$	$N \to L$
3	$K \to O$	$\varnothing$	$M \to O$	$K \to N$
4	$\varnothing$	$O \to K$	$O \to M$	$N \to K$
5	$L \to P$	$P \to L$	$P \to P$	$L \to L$
6	$\varnothing$	$\varnothing$	$O \to O$	$N \to N$
7	$K \to M$	$M \to K$	$M \to M$	$K \to K$

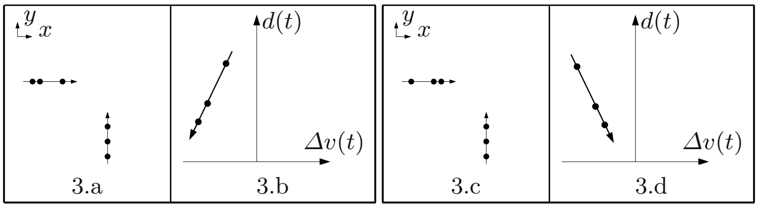

Fig. 3. One object getting close to the other

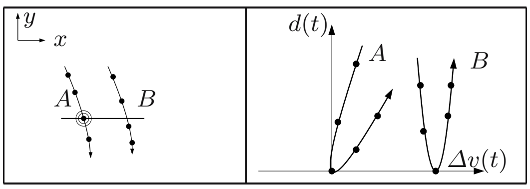

Fig. 4. Two points that touch a line with different behaviours

Figure 4 introduces an example where two target points cross a line with different behaviours. Point A stops on the line before moving away whereas point B crosses the line only, change which is apparent in the relative view only.

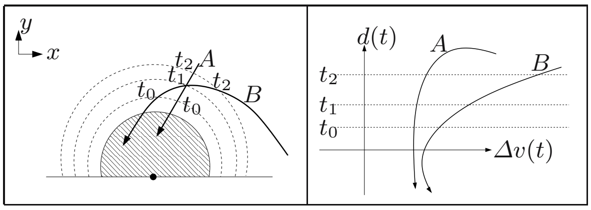

Fig. 5. Two points that enter into a region with different behaviours

Figure 5 presents another example of two target points with respect to an origin region. The distance between the region and the point, at t_0, t_1 and t_2, are the same for two target objects but the relative speeds are different. Speed differences are highlighted by the relative view in Figure 5.2.

4 Conclusion

Recent developments of database structures and languages for the modelling of moving points offer many opportunities for exploratory interfaces that will characterize the semantics exhibited by the underlying processes revealed by these trajectories. This paper introduces a qualitative representation of point trajectories where a relative-based view constitutes an alternative to the conventional Euclidean representation of space. The model is based on two trajectory primitives: relative speed and distance, that are commonly used and perceived as the basic relative constituents of a moving object in space. A complete set of orthogonal dynamic processes is identified, it characterizes the semantics exhibited by a moving point with respect to target points, lines and regions. Such a model complements the cartographical view of space and permits the identification and distinction of the spatio-temporal processes that characterize the behaviour of moving points in a two-dimensional space. The research developed so far is preliminary, further work concerns integration of additional spatial properties such as orientation and acceleration, and implementation of a prototype for the monitoring and analysis of mobile trajectories. We believe that this modelling approach can be applied to several application domains such as traffic monitoring in air, ground and sea.

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